

Three-mode opto-acoustic parametric interactions with coupled cavity

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Abstract

We theoretically analyze three-mode opto-acoustic parametric interactions in a coupled Fabry-Perot cavity, where one acoustic mode interacts with two optical modes. We show explicitly that extra degrees of freedom in a coupled cavity allow explorations of both parametric instability and cooling regimes with high parametric gain in a single table top experiment. This work can motivate experimental realizations of the three-mode parametric instability which might be an issue in next-generation gravitational-wave detectors with high optical-power cavities, helping in the development of better models, and in developing techniques for controls. In addition, we show that the same scheme can be implemented in the resolved-sideband acoustic-mode cooling.

I. INTRODUCTION

Parametric interactions have wide applications and arouse great interests in various fields of physics, from high-sensitivity transducers to low-noise amplifiers and optical parametric oscillators. Two-mode parametric interactions have been used to cool acoustic modes of mechanical oscillators. In Ref. [1], the authors cooled the normal mode of a 1.5 tonne Nb bar down to 5mK by using a microwave parametric transducer. With the same principle, a high-frequency acoustic mode of a nano-mechanical oscillator is cooled near its quantum ground state through coupling to a superconducting single-electron transistor [2]. More recently, by coupling mechanical resonators to optical cavities, various table top experiments have demonstrated significant cooling of acoustic modes [3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13]. These experiments show great potential of achieving the quantum ground state of macroscopic mechanical oscillators, which would be a breakthrough in both theoretical and experimental physics.

Three-mode opto-acoustic parametric interactions were first investigated by Braginsky *et al.* [14, 15] in the context of high optical-power Fabry-Perot cavities for interferometric gravitational-wave detectors. It was shown that three-mode interactions led to a risk of parametric instability (PI) in which the amplitude of acoustic modes of test masses could grow exponentially, thus undermining sensitivities of the detectors. The cause of parametric instability is the radiation pressure mediated coupling between optical modes and acoustic modes. It can occur if the shape of high-order cavity modes, hereafter denoted by TEM_{mn} , have a substantial overlap with that of acoustic modes and simultaneously if the mode gap between TEM_{mn} and TEM_{00} is equal to acoustic-mode frequency up to an error of the linewidth of cavity modes. Further theoretical studies have followed up this pioneering work. Zhao *et al.* [16] extended their analysis and took into account 3D structures of the optical and acoustic modes. Ju *et al.* [17] further considered the overall contributions from multiple optical modes, showing that multiple interactions increase the risk of instability. Gurkovsky *et al.* [18] analyzed PI in a signal-recycled (SR) interferometer and shows that the chance of PI was reduced due to the narrow linewidth of SR interferometer. Additionally, many ideas have been proposed to prevent PI, e.g., changing the radius of curvature (RoC) of the mirror [16], using an optical spring tranquilizer [19] or a ring damper [20]. In the light of the above predictions, it is important to develop experimental techniques for their in-

vestigations. In recent progress, the University of Western Australia group demonstrated three-mode interactions in an 80m-long Fabry-Perot cavity [21]. They used a compensation plate as a thermal lens to tune the effective RoC of the mirror, which changes the Gouy phase or equivalently the mode gap, such that the resonant condition mentioned is satisfied. By capacitively exciting an acoustic mode to interact with the TEM_{00} mode, they observed resonance of a high-order optical mode at the predicted frequency. This experiment confirmed the principle of three-mode interactions, but due to low optical power and small overlap, it had not yet achieved self-sustained parametric instability or cooling. In this paper, we propose an alternative method of tuning the Gouy phase. It was first introduced by Mueller [22] as a means of designing a stable recycling cavity for next-generation gravitational-wave detectors. By adding a mirror and a lens onto a single cavity, which forms a coupled cavity, one can obtain any desired Gouy phase shift simply by adjusting relative positions of these optical components. In following sections, we will show explicitly that a table top experiment can be set up, allowing observations of three-mode interactions with high parametric gain. This can help us better understand possible parametric instability in interferometric gravitational-wave detectors, and can also be applied, in general, as a design concept for opto-acoustic parametric amplifiers, specifically to the resolved-sideband cooling [23, 24] of acoustic modes.

This paper is organized as follows: In Sec. II, we will review the theory of three-mode opto-acoustic interactions. In Sec. III, we will analyze three-mode interactions in a coupled cavity. Finally, we will summarize our results in Sec IV.

II. REVIEW OF THREE-MODE OPTO-ACOUSTIC PARAMETRIC INTERACTION THEORY

For the coherence of this article, we will briefly review the theory of three-mode opto-acoustic parametric interactions. They can be understood qualitatively in both quantum and classical pictures. In the former picture, we can treat the Fabry-Perot cavity as a multi-level quantum system occupied by photons, while the mechanical oscillator (end mirror) consists of phonon (acoustic) modes in different energy eigenstates. For simplicity, we only consider one phonon mode, which represents the situation in this proposed scheme. Initially, the photons stay in one eigenstate (the TEM_{00} mode). Resonant amplification occurs when

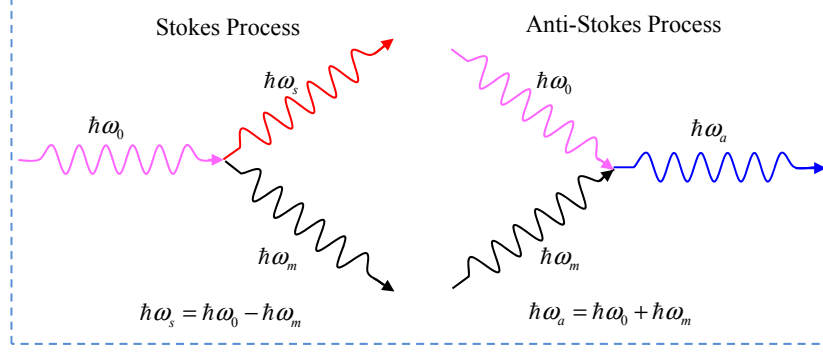


FIG. 1: Parametric interactions between photons and phonons. In the Stokes process, the photon is down converted into a photon with lower energy and at the same time one phonon is created. This process will transfer energy from the optical field into the mechanical degree of freedom, thus amplifying the acoustic mode. In the Anti-Stokes process, the photon is scattered into a higher-frequency photon, accompanied by annihilation of one phonon. In this process, the mechanical oscillation is damped. Classically, these two processes simply correspond to modulations of light by mechanical oscillations into two sidebands at frequencies $\omega_0 \pm \omega_m$.

the photons are scattered by the phonons into another eigenstate (the TEM_{mn} mode). The phonons can absorb energy from the photons (conventionally known as a Stokes process in Raman scattering), causing instability. Alternatively, they can release energy into the optical field (Anti-Stokes process), which is also called cooling in the literature. We show both processes schematically in Fig. 1. In the classical picture [14], mechanical oscillations modulate the carrier frequency ω_0 into two sidebands. The lower sideband corresponds to the Stokes mode with frequency $\omega_s = \omega_0 - \omega_m$ and the upper sideband is the Anti-Stokes mode with $\omega_a = \omega_0 + \omega_m$. If they match the frequency of TEM_{mn} ω_1 , higher-order optical mode will be excited. In turn, these resonant optical modes TEM_{mn} and TEM_{00} will exert a radiation pressure on the mechanical oscillator at the beating frequency $|\omega_1 - \omega_0| = \omega_m$. Depending on the phase, this force will either amplify or damp the motion of the oscillator. To achieve good coupling, a substantial spatial overlap between the acoustic mode and the TEM_{mn} mode is required.

To quantify the interactions, we follow the formalism in Ref. [14]. In the paper, they introduced parametric gain \mathcal{R} to quantify the strength of three-mode interactions, which is

defined as,

$$\mathcal{R} = \frac{4I_c Q_m}{mcL\omega_m^2} \left[\frac{Q_s \Lambda_s}{1 + (\Delta\omega_s/\gamma_s)^2} - \frac{Q_a \Lambda_a}{1 + (\Delta\omega_a/\gamma_a)^2} \right]. \quad (1)$$

Here subscript s denotes Stokes mode and a for Anti-Stokes mode; L is the total length of the cavity; I_c is the intra-cavity power; m is the mass and Q_m is the mechanical quality factor. The detunings $\Delta\omega_{s(a)} \equiv |\omega_{s(a)} - \omega_1|$ and $\gamma_{a(s)}$ is decay rates of optical modes. Overlapping factor $\Lambda_{a(s)}$ is given by,

$$\Lambda_{a(s)} = \frac{V(\int f_0(\vec{r}_\perp) f_{a(s)}(\vec{r}_\perp) u_z d\vec{r}_\perp)^2}{\int |f_0|^2 d\vec{r}_\perp \int |f_{a(s)}|^2 d\vec{r}_\perp \int |\vec{u}|^2 dV}, \quad (2)$$

where $f_{0,a,s}$ is optical-mode shapes; u_z is the component of \vec{u} normal to the mirror surface; The integrals $\int d\vec{r}_\perp$ and $\int dV$ correspond to integration over the mirror surface and volume V respectively. For simplicity, we assume that $\Lambda_{s(a)} \sim 1$, which can be achieved in real experiments. In the tuned case that $\Delta\omega_{s(a)} = 0$, \mathcal{R} can be simply written as

$$\mathcal{R} = \pm \frac{4I_c Q_m Q_{s(a)}}{mcL\omega_m^2}, \quad (3)$$

with $+$ for the Stokes process and $-$ for the Anti-Stokes process. The resulting decay rate of the acoustic mode γ'_m can be written as

$$\gamma'_m = \frac{1}{2} \left[(\gamma_{s(a)} + \gamma_m) - \sqrt{(\gamma_{s(a)} - \gamma_m)^2 + 4\mathcal{R}\gamma_{s(a)}\gamma_m} \right]. \quad (4)$$

Since usually $\gamma_{a(s)} \gg \gamma_m$, then

$$\gamma'_m \approx (1 - \mathcal{R})\gamma_m. \quad (5)$$

When $\mathcal{R} = 0$, we obtain the trivial case $\gamma'_m = \gamma_m$. For the Stokes process with $\mathcal{R} > 0$ (positive gain), the decay rate of the acoustic mode decreases and the mechanical oscillation will be amplified. PI happens if $\mathcal{R} > 1$, where we have $\gamma'_m < 0$. For the Anti-Stokes process with $\mathcal{R} < 0$ (negative gain), the acoustic mode will be cooled as the effective decay rate increases.

III. THREE-MODE INTERACTIONS WITH A COUPLED CAVITY

In this section, we will discuss how to explore three-mode interactions using a coupled cavity. To make our analysis close to realistic experiments, we consider a torsional acoustic mode with frequency ~ 1 MHz interacting with the optical TEM_{10} and TEM_{00} modes. The

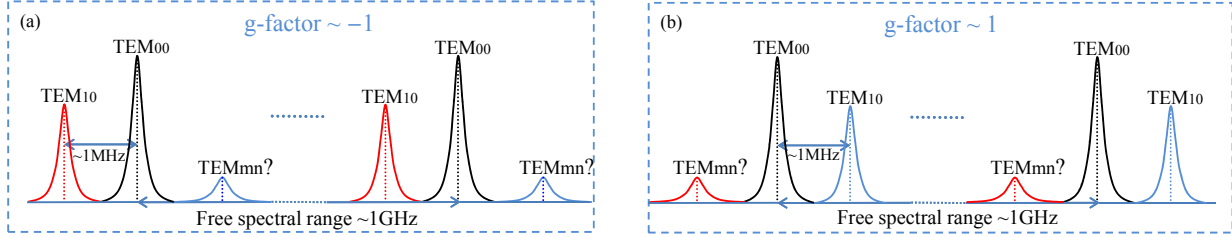


FIG. 2: The optical modes of single Fabry-Perot cavity with length ~ 10 cm. The panel (a) shows the mode distribution for a near-concentric cavity with g -factor ~ -1 which suits for observing PI, while panel (b) is the near-planar case with g -factor ~ 1 which suits for cooling experiment. In both cases, there are no symmetric modes on the opposite side of the TEM₀₀ mode because the higher order mode TEM _{mn} marked with ‘?’ are highly lossy due to diffraction losses. This is preferred for experimental realizations of three-mode interactions because we know from Eq. (1) that any symmetric mode on the opposite side of the TEM₀₀ mode will reduce the absolute value of parametric gain. However, both cavities are marginally stable and very susceptible to misalignment.

configuration is chosen because MHz can be easily achieved in a mm-scale structure and the torsional mode has a large spatial overlap with the TEM₁₀ mode.

To begin with, let us consider a single Fabry-Perot cavity to see why a coupled cavity is necessary. The free spectral range of a single cavity with length ~ 10 cm is approximately 1GHz. Therefore, as shown in Fig. 2, one has to build either a near-planar or near-concentric cavity to obtain a desired mode gap around 1MHz between TEM₁₀ and TEM₀₀. For both cases, the cavity is marginally stable and susceptible to misalignment. It is also difficult to get accesses to both instability and cooling regimes in a single setup. Creating a coupled Fabry-Perot cavity solves these problems. As we will show later, the resulting scheme is stable. Additionally, we can easily tune between instability and cooling regimes. The coupled cavity is showed schematically in Fig. 3. It is similar to the configuration of power or signal recycling interferometers [25, 26] when one considers either the common mode or the differential mode. The field dynamics can be easily obtained as shown in Ref. [27] by treating the sub-cavity as an effective mirror with frequency and mode-dependent transmissivity and reflectivity. Specifically, the effective transmissivity t_{01} is

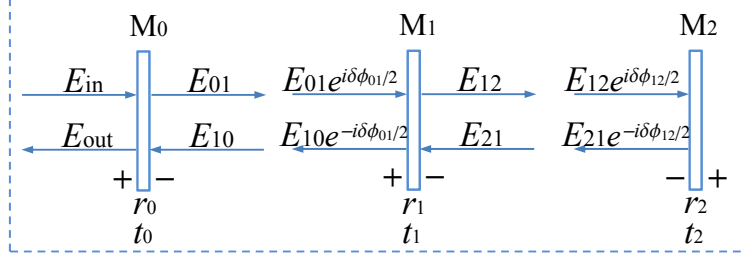


FIG. 3: The optical fields of the coupled cavity. Here $\delta\phi_{01,12}$ are the round-trip phase shift of light in the sub-cavity (formed by M_0 and M_1) and main cavity (formed by M_1 and M_2) respectively. We use the convention that the mirrors have minus reflectivity on the side with a coating layer.

$$t_{01} \equiv \frac{E_{12}}{E_{in}} = \frac{t_0 t_1}{1 + r_0 r_1 e^{i\delta\phi_{01}}}, \quad (6)$$

and the effective reflectivity r_{10} is given by

$$r_{10} \equiv \frac{E_{12}}{E_{21}} = -r_1 - \frac{t_1^2 r_0 e^{i\delta\phi_{01}}}{1 + r_0 r_1 e^{i\delta\phi_{01}}}. \quad (7)$$

The corresponding E_{12} inside the main cavity can be written as

$$E_{12} = \frac{E_{in} t_{01}}{1 + r_{10} r_2 e^{i\delta\phi_{12}}} = \frac{E_{in} t_{01}}{1 - |r_{10}| r_2 e^{i[\arg(r_{10}) + \delta\phi_{12} + \pi]}}. \quad (8)$$

The resonance occurs when the phase factor in Eq. (8) is equal to $2n\pi$, which critically depends upon the phase angle of the effective reflectivity, namely $\arg(r_{10})$. Specifically, when the TEM_{00} mode resonates inside the main cavity, which requires that $\delta\phi_{01}^{\text{TEM}_{00}} = \delta\phi_{12}^{\text{TEM}_{00}} = 2n\pi$, the phase shift of the TEM_{10} mode $\delta\phi_{ij}^{\text{TEM}_{10}}$ is

$$\delta\phi_{ij}^{\text{TEM}_{10}} = \frac{2L_{ij}}{c} \Delta\omega - 2\Phi_g^{ij} + 2n'\pi, \quad ij = 01, 12, \quad (9)$$

where $\Delta\omega \equiv \omega_1 - \omega_0$ is the mode gap between TEM_{10} and TEM_{00} ; Φ_g is the Gouy phase and n, n' are integers. In order to satisfy the resonant condition for three-mode interactions, we need to adjust $\delta\phi_{01}^{\text{TEM}_{10}}$, which changes $\arg(r_{10})$, such that $\Delta\omega = \pm\omega_m$. To achieve this, one obvious way is to change the length of sub-cavity L_{01} , but this turns out to be impractical due to a small tuning range. An alternative and more practical approach, as shown in Fig. 4, is to add another lens or concave mirror inside the sub-cavity to tune the Gouy phase Φ_g^{01} . The resulting scheme is similar to the proposed stable recycling cavity for next-generation gravitational-wave detectors [22]. With the additional lens, the Gaussian

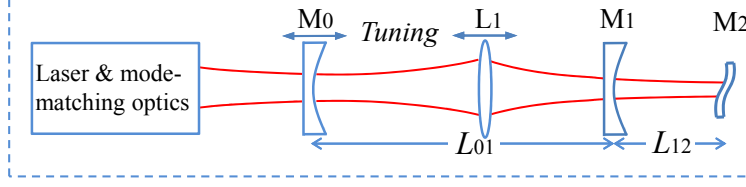


FIG. 4: The optical layout for the table top experiment, where a 1 MHz micro torsional oscillator (M_2) interacts with the optical TEM_{10} mode and the TEM_{00} mode. By tuning the positions of mirror M_0 and lens L_1 , we can continuously change the frequency of TEM_{10} . If the losses in L_1 were an issue, it could easily be replaced by a concave mirror.

beam gets focused inside the sub-cavity. Since the Gouy phase changes almost from $-\frac{\pi}{2}$ to $\frac{\pi}{2}$ within one Rayleigh range around the waist, one can easily obtain a desired $\delta\phi_{01}^{TEM_{10}}$ simply by adjusting the position of M_0 near the waist. This might lead to problems with power density due to the small waist size, but for the table top experiment we consider here, the power density is quite low.

The corresponding Φ_g^{01} with an additional lens can be derived straightforwardly by using the ray transfer relation for a Gaussian beam, which is given by

$$q' = \frac{fq}{f - q}. \quad (10)$$

Here f is the focal length of L_1 ; $q^{(l)} \equiv z^{(l)} + iz_R^{(l)}$; z is the displacement relative to the waist; z_R is the Rayleigh range and superscript $'$ denotes quantities after the lens. This dictates

$$z' = \frac{f(zf - z^2 - z_R^2)}{(f - z)^2 + z_R^2}, \quad (11)$$

$$z_R' = \frac{z_R f^2}{(f - z)^2 + z_R^2}. \quad (12)$$

The resulting Gouy phase at any point is given by

$$\Phi_g(z) = \begin{cases} \arctan(z/z_R), & z < z_L \\ \arctan[(z - z_L + z_L')/z_R'] + \arctan(z_L/z_R) - \arctan(z_L'/z_R'), & z \geq z_L \end{cases} \quad (13)$$

where $z_L^{(l)}$ is the position of L_1 relative to the waist. Gouy phase Φ_g^{01} is the difference between wavefront at M_0 and M_1 , namely

$$\Phi_g^{01} = \Phi_g(z_{M_0}) - \Phi_g(z_{M_1}), \quad (14)$$

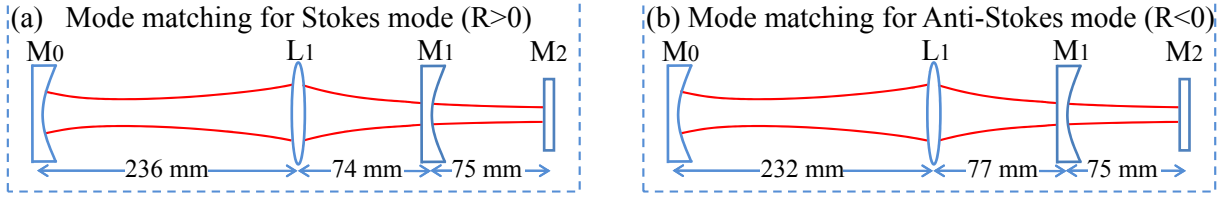


FIG. 5: The mode matching for the positive gain and negative gain by adjusting the relative position of M_0 and L_1 . Only small adjustment is needed to tune from one case to another.

where z_{M_0} and z_{M_1} are the positions of M_0 and M_1 relative to the waist respectively. Therefore, by adjusting the positions of M_0 and L_1 as shown in Fig. 4, we can continuously tune Φ_g^{01} such that $\Delta\omega = \omega_m$.

Equations (6) to (14) provide the design tools of the coupled cavity for three-mode interactions. To realize the experiment, we first need to design the main cavity and specify L_{12} , ω_m , the RoCs of M_0, M_1, M_2 and the focal length of L_1 . From Eq. (8) and Eq. (9), we can find out the required $\arg(r_{01})$ which gives the right mode gap between TEM_{10} and TEM_{00} . This will give us one constraint. Combining with the requirement of mode matching to M_0 , we can fix two degrees of freedom of the system, namely the positions of M_0 and L_1 . To demonstrate this principle explicitly, we present a solution that is close to a realistic experimental setup. We assume the following,

$$L_{12} = 75 \text{ mm} \quad \omega_m = 1 \text{ MHz} \quad f = 100 \text{ mm}$$

$$R_0 = 500 \text{ mm} \quad r_0 = \sqrt{0.999} \quad A_0 = 500 \text{ ppm}$$

$$R_1 = 100 \text{ mm} \quad r_1 = \sqrt{0.9} \quad A_1 = 500 \text{ ppm}$$

$$R_2 = \infty \text{ mm} \quad r_2 = \sqrt{0.9995} \quad A_2 = 500 \text{ ppm}.$$

Here $R_i (i = 0, 1, 2)$ are RoCs; r_i denotes the amplitude reflectivity; t_i is the amplitude transmissivity and A_i is the optical loss which satisfy $r_i^2 + t_i^2 + A_i = 1$ ($i = 0, 1, 2$).

The results of mode matching for both positive (instability) and negative gain (cooling) configurations are shown in Fig. 5. In Fig. 6, we show the mode gap between TEM_{10} and TEM_{00} as a function of the position of M_0 relative to L_1 and the position of L_1 relative to M_1 . In this particular case, the dependence is almost linear with a slope $\sim 2 \text{ mm/MHz}$ for both panels. This indicates that to tune within a cavity linewidth $\sim 0.1 \text{ MHz}$, it requires

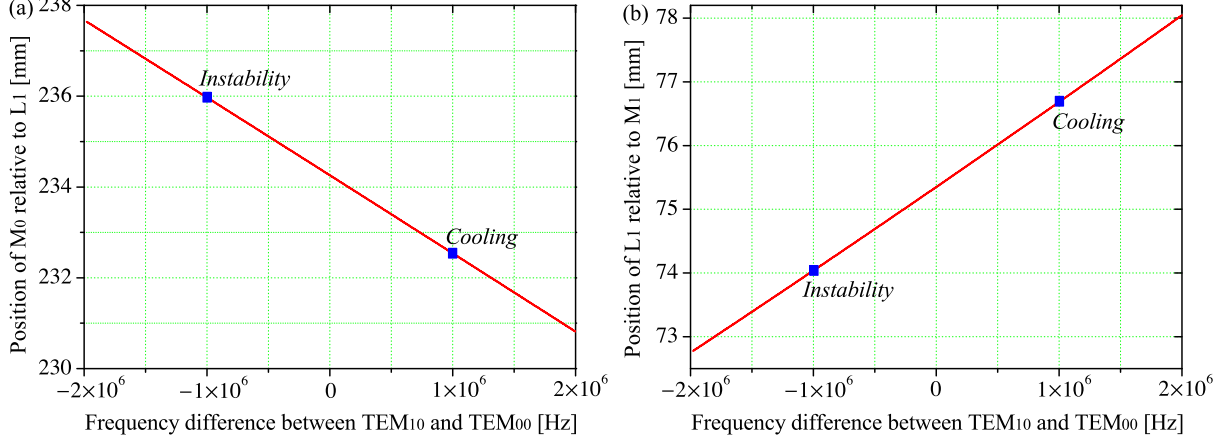


FIG. 6: Panels (a) and (b) show the mode gap between TEM_{10} and TEM_{00} as a function of position of M_0 relative to L_1 and position of L_1 relative to M_1 respectively. The dots in both figures are the situations considered in Fig. 5. Clearly, we can tune between the instability and cooling regimes continuously.

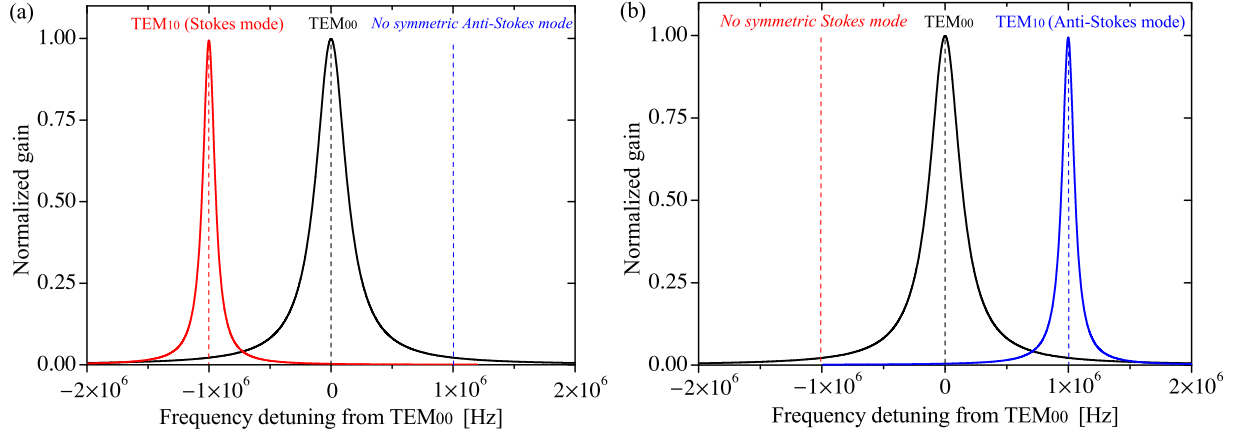


FIG. 7: The normalized gain of the TEM_{00} mode and the TEM_{10} mode in the positive and negative gain configurations. The mode gap is equal to $\omega_m \sim 1\text{MHz}$, which fulfils the resonant condition for the three-mode opto-acoustic interactions. Here we simply assume that the size of the mirrors is infinite so the quality factor of the TEM_{10} mode is solely due to optical losses such as absorbtion. This assumption is reasonable when the mode number is small. Given the specifications in the main text, $Q_a \approx Q_s = 2.4 \times 10^9$ and $\omega_m/\gamma_a < 1$. Therefore, it can be implemented in the resolved-sideband cooling. (a) The TEM_{10} mode is 1 MHz below the TEM_{00} mode; (b) The TEM_{10} mode is 1 MHz above the TEM_{00} mode.

the mirror position to be adjusted within several $100\mu\text{m}$, which can be achieved easily. Therefore, we can continuously tune between instability and cooling regimes. Fig. 7 shows the resulting resonance curves for both cases with the corresponding mode matching shown in Fig. 5. The corresponding mode gap between the TEM_{10} and TEM_{00} modes is equal to $\omega_m \sim 1\text{MHz}$. More importantly, there is no symmetric mode on the opposite side of the TEM_{00} mode, whose presence could contribute a parametric gain with the opposite sign, thereby suppressing the overall effects. The absolute value of parametric gain \mathcal{R} could be larger than 1, if we further assume that the intra-cavity power I_c is 100 mW, $Q_m = 10^6$, the mass of the oscillator $m = 1\text{ mg}$ and the wavelength of light is 1064 nm. Since the cavity is in the resolved-sideband regime where the cavity linewidth is much smaller than the mechanical frequency [23], this configuration can also be applied in the resolved-sideband cooling of acoustic modes, which is less susceptible to quantum noise. Although the quantum noise analysis in Refs. [23, 24, 28] only discusses a two-mode system, their results can be extended to the three-mode system by viewing the TEM_{00} mode as the carrier light. This can be justified by writing down the Hamiltonian of this system, which is

$$\hat{H} = \frac{\hat{p}^2}{2m} + \frac{1}{2}m\omega_m^2\hat{x}^2 + \hbar\omega_0\hat{a}^\dagger\hat{a} + \hbar\omega_1\hat{b}^\dagger\hat{b} + \hbar G_0\hat{x}(\hat{a}^\dagger\hat{b} + \hat{b}^\dagger\hat{a}) + \hat{H}_{\text{drive}}, \quad (15)$$

Here the first two terms represent the energy of the acoustic mode, where \hat{x} is displacement and \hat{p} is momentum which satisfy the commutation relation $[\hat{x}, \hat{p}] = i\hbar$; \hat{a} and \hat{b} are the annihilation operators of the TEM_{00} and TEM_{10} modes respectively; the fifth term quantifies the interactions between the optical modes and the acoustic mode with the coupling constant $G_0 \equiv \Lambda_{a,s}\omega_0/L$; the last term corresponds to external driving. Since only the TEM_{00} mode is pumped externally, the interaction term can be linearized as

$$\hat{H}_I \approx \hbar G_0\hat{x}(\bar{a}^*\hat{b} + \bar{a}\hat{b}^\dagger), \quad (16)$$

which is the same interaction considered in Ref. [23, 28] after linearizing. Therefore, this three-mode Hamiltonian can be mapped into an effective two-mode Hamiltonian by replacing \hat{a} with its classical amplitude \bar{a} . All the conclusions reached in the two-mode case are also valid here. The only difference is that the carrier light TEM_{00} is also on resonance rather than far detuned in the two-mode case [23, 28]. Therefore, we can achieve the resolved-sideband limit without compromising intra-cavity optical power. What's more, the resonant condition $\omega_1 - \omega_0 = \omega_m$ here is equivalent to the frequency detuning equal to ω_m in the two-mode

case. This optimizes the energy transfer from the acoustic mode to the optical fields in the resolved-sideband limit [23, 28].

IV. CONCLUSION

We have shown that a table top experiment based on a coupled cavity can be set up to realize self-sustained three-mode opto-acoustic parametric interactions. Small adjustments of the positions of the optical components enable us to continuously tune between instability and cooling regimes. This relative simple scheme can be applied to design opto-acoustic amplifiers and experimentally investigate the three-mode parametric instability, a possible issue of next-generation gravitational-wave detectors with high optical-power cavities. Besides, it can also be used to realize the resolved-sideband cooling of acoustic modes with three-mode interactions.

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- [1] D. G. Blair, E. N. Ivanov, M. E. Tobar, P. J. Turner, F. van Kann and I.S. Heng, Phys. Rev. Lett. **74**, 1908 (1995);
 - [2] A. Naik, O. Buu, M. D. LaHaye, A. D. Armour, A. A. Clerk, M. P. Blencowe and K. C. Schwab, Nature **443**, 14 (2006);

- [3] S. Gigan, H. R. Böhm, M. Paternostro, F. Blaser, G. Langer, J. B. Hertzberg, K. C. Schwab, D. Bäuerle, M. Aspelmeyer and A. Zeilinger, *Nature* **444**, 67 (2006);
- [4] O. Arcizet, P. F. Cohandon, T. Briant, M. Pinard and A. Heidmann, *Nature* **444**, 71 (2006);
- [5] D. Kleckner and D. Bouwmeester, *Nature* **444**, 75 (2006);
- [6] A. Schliesser, P. DelHaye, N. Nooshi, K. J. Vahala, and T. J. Kippenberg, *Phys. Rev. Lett.* **97**, 243905 (2006);
- [7] T. Corbitt, Y. Chen, E. Innerhofer, H. Muller-Ebhardt, D. Ottaway, H. Rehbein, D. Sigg, S. Whitcomb, C. Wipf and N. Mavalvala, *Phys. Rev. Lett.* **98**, 150802 (2007);
- [8] T. Corbitt, C. Wipf, T. Bodiya, D. Ottaway, D. Sigg, N. Smith, S. Whitcomb, and N. Mavalvala, *Phys. Rev. Lett.* **99**, 160801 (2007);
- [9] A. Schliesser, R. Riviere, G. Anetsberger, O. Arcizet, T. J. Kippenberg. arXiv: 0709.4036v1 [quant-ph] (2007);
- [10] M. Poggio, C. L. Degen, H. J. Mamin, and D. Rugar, *Phys. Rev. Lett.* **99**, 017201 (2007);
- [11] J. D. Thompson, B. M. Zwickl, A. M. Jayich, F. Marquardt, S. M. Girvin and J. G. E. Harris, *Nature* **452**, 72 (2008);
- [12] C. M. Mow-Lowry, A. J. Mullavey, S. Goßler, M. B. Gray and D. E. McClelland, *Phys. Rev. Lett.* **100**, 010801 (2008);
- [13] S. W. Schemm, C. Zhao, L. Ju, D. G. Blair and P. Willems, *Phys. Rev. A* **77**, 013813 (2008);
- [14] V. B. Braginsky, S. E. Strigin and S. P. Vyatchanin, *Phys. Lett. A* **287**, 331 (2001);
- [15] V. B. Braginsky, S. E. Strigin and S. P. Vyatchanin, *Phys. Lett. A* **305**, 111 (2002);
- [16] C. Zhao, L. Ju, J. Degallaix, S. Gras and D. G. Blair, *Phys. Rev. Lett.* **94**, 121102 (2005);
- [17] L. Ju, S. Gras, C. Zhao, J. Degallaix and D. G. Blair, *Phys. Lett. A* **354**, 360 (2006);
- [18] A. G. Gurkovsky, S. E. Strigin, S. P. Vyatchanin, *Phys. Lett. A* **362**, 91 (2007);
- [19] V. B. Braginsky and S. P. Vyatchanin, *Phys. Lett. A* **293**, 228 (2002);
- [20] S. Gras, D. G. Blair and L. Ju, *Phys. Lett. A* **372**, 1348 (2008);
- [21] C. Zhao, L. Ju, Y. Fan, S. Gras, B. J. J. Slagmolen, H. Miao, P. Barriga, D. G. Blair, D. J. Hosken, A. F. Brooks, P. J. Veitch, D. Mudge, and J. Munch, *Phys. Rev. A* **78**, 023807 (2008);
- [22] G. Mueller, LIGO document: G070441-00-R;
- [23] F. Marquardt, J. P. Chen, A. A. Clerk, and S. M. Girvin, *Phys. Rev. Lett.* **99**, 093902 (2007);
- [24] I. Wilson-Rae, N. Nooshi, W. Zwerger, and T. J. Kippenberg, *Phys. Rev. Lett.* **99**, 093901

- (2007);
- [25] B. J. Meers, Phys. Rev. D **38**, 8 (1988);
- [26] A. Buonanno and Y. Chen, Phys. Rev. D **64**, 042006 (2001);
- [27] M. Rakhmanov, *Dynamics of Laser Interferometric Gravitational Wave Detectors* (2000)
(PhD Thesis);
- [28] C. Genes, D. Vitali, P. Tombesi, S. Gigan and M. Aspelmeyer, Phys. Rev. A **77**, 033804
(2008).